XIX.—*Experimental Researches on the Lifting Power of the Electro-Magnet. Part II. Temperature Correction; Effects of Spirals and Helices. By the* Rev. T. R. Robinson, D. D., *President of the Royal Irish Academy, and Member of other Scientific Societies.*

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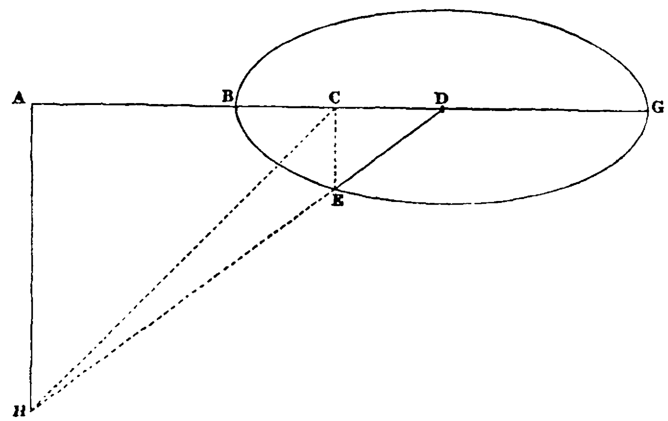
In my former communication on the subject, I examined the relation between the lifting power of the electro-magnet and the force of the current which excites it; and shewed that the first increases much more slowly than the second, so that it cannot pass a limit which depends on the size of the magnet by any assignable amount of current force. But besides the magnitude of that force, the magnet's power depends even more on the number and distribution of the spires of its helices; we can dispose of a very restricted amount of current. The most advantageous mode of employing a given battery is when its internal and external resistance are equal, its action therefore $= \frac{E}{2R}$. This for the Grove's which I use, exposing 19 inches of platinum, is 6 of my units; and for my Callan's of 90 square inches is 14.5, the last of which would only excite my magnet with a single spire to one-sixtieth of its maximum. But however we increase the number of spires, they have all an exciting power; and if this acted equally for each on the magnet, the effect of the current might be increased without limit. It is true that the increased resistance would require a larger battery, but this can always be commanded. But this equality of action does not exist; the exterior spires act more feebly on account of their greater distance, and those at a distance from the polar surfaces exert little influence on them, both from distance and obliquity of force; and secondly, though they do excite fully the parts near their plane, yet the magnetism developed there is

greatly weakened by the induction which transmits it to the poles. As far as I know, these disturbing causes have not been studied ; and I hope that the following results will not be without their value to the construction as well as theory of the electro-magnet.

The electro-dynamic laws, discovered by Ampere, Biot, and others who have followed in their steps, may seem sufficient to establish its theory ; and it is desirable to ascertain whether they succeed in doing so, for the simple case of a circular or helical current.

Let BEG be a differential slice of the magnet, E one of its elements, H a current element below its plane, whose direction is perpendicular to the plane HAG, the power of H to attract and, as we may infer, to magnetise E is, as its magnitude dc, as its energy, μF (F being the force of the current), as the inverse square of the distance HE, and as the cosine of the angle CHE; or putting $HA = z$, $AC = x$, $CE = y$, $DE = r$, $AD = b$, and $BDE = \theta$, the magnetism of E,



$$dM' = \frac{\mu F . dc . E \sqrt{(x^2 + z^2)}}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}.$$

A difficulty occurs here however. The polarity given to the element E is such, that its axis is perpendicular to HE; but, as in the case before us, the current surrounds the magnet, it is evident that unless $z = 0$, each portion of it must produce a different axis. Is this physically possible in the molecules of iron ? If so, their polarities must be very irregularly distributed. Or do the axes coalesce into one resultant, whose intensity may be estimated by the composition of forces ? This seems most probable ; and, therefore, assuming it, and resolving dM' in the direction of z,

$$dM = \frac{\mu F E dc \times x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}.$$

As the magnet has cylindric arms, BEG is a circle; E, therefore, $= rdrd\theta dl$, transporting the origin to the centre, we have

$$dM = \frac{\mu F dcdl\,(b - r\cos\theta)\,.\,rdrd\theta}{(b^2 + r^2 + z^2 - 2br\cos\theta)^{\frac{3}{2}}}. \tag{1}$$

Integrating this for θ from 0 to 2π, and for r from 0 to r', we obtain the magnetic force of a slice of the magnet whose thickness $= dl$, due to the action of H.

This, however, assumes that each molecule is susceptible of magnetism up to the full influence of the current on it, which can scarcely be the fact. Those nearest the helix being most excited, must tend to induce polarities opposite to their own on those next within, on which also the direct action is less energetic; and we may, therefore, expect to find a zone of intense magnetism succeeded by one weaker, null, or even reversed, followed by a series of similar alternations. This does occur in compound magnets to a great extent; and is manifest in those experiments of Plücker, which prove that a mass of iron is less attracted than filings of the same metal, and these less than powder of iron, more sparsely distributed by being diffused through lard. Of course the same inductive interference occurs in the case before us; but we know too little of its laws to be able to introduce it into the calculation.

The first integral belongs to a class which presents considerable difficulty when its modulus is so near unity, as must be the case with the innermost spires; and among the methods of approximation which have been devised by Euler and Legendre, none, on the whole, are as convenient for my purpose as the common development by the Binomial theorem. Let $b^2 + r^2 + z^2 = u^2$, $rdr = udu$; and expanding $\left(1 - \frac{2br\cos\theta}{u^2}\right)^{-\frac{3}{2}}$, and omitting odd powers of $\cos\theta$, because the terms introduced by their integration vanish between the limits, we obtain

$$dM = \frac{\mu F dcdldud\theta}{u^2}\left\{\begin{array}{l} b + A \times \frac{2br^2}{u^2}\left(\frac{5}{4} \times \frac{2b^2}{u^2} - 1\right)\cos^2\theta \\ \quad + C \times \frac{(2b^3)r^4}{u^6}\left(\frac{9}{8} \times \frac{2b^2}{u^2} - 1\right)\cos^4\theta \\ \quad + E \times \frac{(2b)^5 r^6}{u^{10}}\left(\frac{13}{12} + \frac{2b^2}{u^2} - 1\right)\cos^6\theta \\ \quad + \quad \&c. \quad \&c. \end{array}\right\}$$

The general term being

$$\frac{3.5\ldots.2n-3.2n-1}{2.4\ldots.2n-4.2n-2}\times\frac{(2b)^{n-1}\times r^n.\cos^n\theta}{u^{2n-2}}\times\left(\frac{2n+1}{2n}\times\frac{2b^2}{u^2}-1\right).$$

When n is even,

$$\int_0^{2\pi}\cos^n\theta d\theta=2\pi\left\{\frac{1.3.5\ldots.n-1..}{1.2.3\ldots.\frac{1}{2}n\ldots.}\times\frac{1}{2^{\frac{n}{2}}}\right\},$$

and, therefore, the integral of the corresponding term is

$$\pi\times\frac{3.5\ldots.2n-3.2n-1}{2.4\ldots.2n-4.2n-2}\times\frac{1.3.5\ldots n-1}{1.2.3\ldots\frac{1}{2}n}\times\frac{2^{\frac{n}{2}}.b^{n-1}.r^n}{u^{2n-2}}\left(\frac{2n+1.b^2}{n.u^2}-1\right),$$

and it is derived from that which precedes it by multiplying the latter by the factor

$$\frac{2n-3.2n-1}{n-2.n}\times\frac{b^2r^2}{u^4}; \tag{2}$$

we thus obtain

$$dM=\mu F.dc.dl.du.\pi\left\{\begin{array}{l}\dfrac{b}{u^2}+\dfrac{3br^2}{2u^4}\left(\dfrac{5b^2}{2u^2}-1\right)+\dfrac{3.5.7.b^3r^4}{2.2.4.u^8}\left(\dfrac{9b^2}{4u^2}-1\right)\\ +\dfrac{3.5.7.9.11.b^5.r^6}{2.2.4.4.6.u^{12}}\left(\dfrac{13b^2}{6u^2}-1\right)+\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ +\dfrac{3.5.7\ldots2n-3.2n-1.b^{n-1}.r^n}{2.2.4\ldots.n-2.n.u^{2n}}\left(\dfrac{2n+1.b^2}{nu^2}-1\right).\end{array}\right.$$

This must now be integrated for r. As $rdr=udu$, integrating by parts,

$$\int\frac{r^ndu}{u^m}=\frac{-r^2}{m-1.u^{m-1}}-\frac{nr^{n-2}}{m-1.m-3.u^{m-3}}-\frac{n.n-2.r^{n-4}}{m-1.m-3.m-5.u^{m-5}},\ \&c.,$$

which must be continued till the exponent of r vanishes. Each value of n gives two sets of terms on account of the factor $\frac{(2n+1)b^2}{n.u^2}-1$; if then we combine the first set of the n^{th} with the second of the $(n+2)^{th}$, we have the sum

$$Pb^{n+1}\times\frac{2n+1}{n}\times$$

$$\left\{\begin{array}{l}-\dfrac{r^n}{2n+1.u^{2n+1}}-\dfrac{nr^{n-2}}{2n+1.2n-1.u^{2n-1}}-\dfrac{n.n-2.r^{n-4}}{2n+1.2n-1.2n-3.u^{2n-3}}-\&c.\\ +\dfrac{r^{n+2}}{n+2.u^{2n+3}}+\dfrac{r^n}{2n+1.u^{2n+1}}+\dfrac{nr^{n-2}}{2n+1.2n-1.u^{2n-1}}+\&c.\end{array}\right\}$$

which destroy each other, except the term

$$\frac{Pb^{n+1} \,.\, 2n+1 \,.\, r^{n+2}}{n \,.\, n+2 \,.\, u^{2n+3}},$$

which belongs to the exponent $n+2$. If the term of $n = T_n$,

$$T_n = \frac{Pb^{n-1} \,.\, r^n}{2n-1 \,.\, u^{2n-1}},$$

and the next,

$$T_{n+2} = T_n \times \frac{2n+1 \,.\, 2n-1 \,.\, b^2 \,.\, r^2}{n \,.\, n+2 \,.\, u^4}, \tag{3}$$

from which the successive terms of the integral (which all vanish when $r=0$) are easily formed. We thus find, calling the section of the magnet $\pi r^2 dl = A$,

$$dM = \mu F A \left\{ \begin{array}{l} \dfrac{b}{2u^3} + \dfrac{1\,.\,3\,.\,5\,.\,b^3 r^2}{2\,.\,2\,.\,4\,.\,u^7} + \dfrac{1\,.\,3\,.\,5\,.\,7\,.\,9\,.\,b^5\,.\,r^4}{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,u^{11}} \\ + \dfrac{1\,.\,3\,.\,5\,.\,7\,.\,9\,.\,11\,.\,13\,.\,b^7\,.\,r^6}{2\,.\,2\,.\,4\,.\,4\,.\,6\,.\,6\,.\,8\,.\,u^{15}} + \&c. \end{array} \right\} \times dc. \tag{4}$$

This converges sufficiently, unless b is nearly $= r$; then, notwithstanding the simplicity of the law of continuation, the computation is tedious. But as soon as n is so large that $\frac{1}{n^2+2n}$ may be neglected, it can be much simplified; for, calling $\frac{4b^2r^2}{u^4} = \rho$, (3) becomes

$$T_{n+2} = T_n \, \rho \times \frac{n}{n+2}; \quad T_{n+4} = T_n \, \rho^2 + \frac{n}{n+4};$$

and x being the number of steps,

$$T_{n+2x} = T_r \, \rho^x + \frac{n}{n+2x}. \tag{5}$$

Even this is too slow; but it enables us to compute x terms *per saltum*. The sum of them is $(m = \frac{1}{2}n)$,

$$T_n \left\{ \frac{\rho m}{m+1} + \frac{\rho^2 m}{m+2} + \frac{\rho^3 m}{m+3} \ldots + \frac{\rho^x m}{m+x} \right\},$$

or developing and arranging according to the powers of $\frac{1}{m}$, and putting

$$\rho + \rho^2 \;.\;.\;.\;. + \rho^x = \frac{\rho}{1-\rho}(1-\rho^x) = A,$$

$$\rho + 2\rho^2 + \;.\;.\;.\;. + x\rho^x = \frac{\rho}{(1-\rho)^2}\{1-(x+1)\,\rho^x + x\rho^{x+1}\} = B.$$

$$\rho+4\rho^2+9\rho^3\ldots+x^2\rho^x=\frac{\rho}{(1-\rho)^3}\{1+\rho-(x+1)\rho^x+(2x^2+2x-1)\rho^{x+1}-x^2\rho^{x+2}\}=C;$$

we have
$$S(T_{n+1}\ldots T_{n+2x})=T_n\left\{A-\frac{B}{m}+\frac{C}{m^2}\right\}, \tag{6}$$

which is sufficient for practice. I take $x=10$ or 20, and thus obtain the value of the integral very rapidly. This must now be integrated for c, which admits of two cases. In the first the current is a circle whose radius $= b$, therefore $\int dc=2\pi b$; and if $S=$ the sum of the series in (4), the action of a single ring or convolution of a spiral, whose plane is perpendicular to the axis.

$$M=\mu F\,.\,A\,.\,S\times 2\pi b. \tag{7}$$

No sensible error can arise from considering it a circle ; and I have computed the following table of the coefficient of $\mu\,.\,F\,.\,A$ for the magnet and spirals which I use, in which $r=1$, and the least value of $b=1.13$.

Table I.

b	$z=0.$	$z=2'.$	$z=4'\cdot0.$	$z=6'\cdot0.$	$z=8'\,0.$	$z=10'\cdot0.$
1·13	4·64861	0·27331	0·05165	0·01904	0·00738	0·00390
	·47328					
1·33	3·17533	0·34107	0·06874	0·02306	0·01019	0·00541
	·65685					
1·53	2·51848	0·40382	0·08722	0·02971	0·01325	0·00703
	·41086					
1·73	2·10762	0·45736	0·10757	0·03720	0·01671	0·00887
	·28624					
1·93	1·82138	0·50204	0·12805	0·04526	0·02048	0·01093
	·20683					
2·13	1·61455	0·53619	0·14628	0·05351	0·02457	0·01313
	·16073					
2·33	1·45382	0·56135	0·16393	0·06206	0·02890	0·01552
	·13127					
2·53	1·32255	0·57794	0·18134	0·07068	0·03349	0·01810
	·10852					
2·73	1·21403	0·58788	0·19831	0·07956	0·03806	0·02076
	·9134					
2·93	1·12269	0·59570	0·21376	0·08832	0·04286	0·02349
	·7785					
3·13	1·04484	0·59226	0·22864	0·09676	0·04762	0·02632
	·6758					
3·33	0·97726	0·58978	0·24042	0·10541	0·05268	0·02941
	·5932					
3·53	0·91794	0·58268	0·25166	0·11340	0·05764	0·03253
	·5194					
3·73	0·86600	0·57465	0·26148	0·12155	0·06261	0·03555
	·4457					
3·93	0·82143	0·56522	0·27009	0·12913	0·06756	0·03870

It appears from the second column of this table that the power of a ring decreases with an increase of its diameter, very rapidly at first, but more slowly afterwards, so that its action continues sensible to a considerable distance. But out of its plane the case is different, the total effect is much less; but if z have any considerable magnitude, it increases with the diameter of the ring.

The case of a spiral is, however, not that of most ordinary occurrence, the wire being generally disposed in a helix. To obtain its effect on the magnet's element A, we substitute for dc in (4) the differential of the helix. In this curve if $e =$ the slope of the wire,

$$z = b\theta \times \tan e, \quad dc = \frac{dz}{\sin e};$$

as, however, the curve is inclined and its induction is in a plane perpendicular to it, dc must be resolved in the direction of its base, and we have

$$dM = \frac{\mu F . A \times dz}{\tan e} \times \left\{ \frac{br^2}{2u^3} + \frac{1 . 3 . 5 . b^3 r^4}{2 . 2 . 4 . u^7} + \&c. \right\}$$

Putting $b^2 + r^2 = t^2$, the integral consists of a series of terms,

$$\frac{\int b^{n-1} dz \times r^n}{(t^2 + z^2)^{\frac{2n-1}{2}}} = b^{n-1} \times r^n z \left\{ \begin{array}{l} \dfrac{1}{2n-3 . t^2 . u^{2n-3}} + \dfrac{2n-4}{2n-3 . 2n-5 . t^4 . u^{2n-5}} \\ \dots\dots\dots\dots\dots\dots\dots \\ + \dfrac{P . 2n-2i}{2n-(2i+1)\, t^2 . u^{2n-(2i+1)}} \end{array} \right\}$$

which vanish with z, and therefore require no constant.

The series terminates when $i = n$, and its last term is

$$Q_n = \frac{r^n . b^{n-1}}{t^{2n-2} . \times u} \left\{ \frac{2 . 4 . 6 \dots 2n-4 . 1}{1 . 3 . 5 \dots . 2n-5 . 2n-3} \right\}.$$

The term preceding this is obtained by multiplying it by $\frac{1 . t^2}{2 . u^2}$; the next by the additional factor $\frac{3 . t^2}{4 . u^2}$, and so on till the last factor is $\frac{2n-5 . t^2}{2n-4 . u^2}$.

Having obtained Q_n, the next term,

$$Q_{n+2} = Q_n \times \frac{2n-2 . 2n}{2n+1 . 2n-1} \times \frac{r^2 b^2}{t^4};$$

and as each term of (4) has a coefficient, whose law of derivation is $\frac{2n+1 \, . \, 2n-1}{n \, . \, n+2}$, we have

$$Q_{n+2} = Q_n \times \frac{n-1}{n+2} \times \frac{4r^2b^2}{t^4},$$

so that the successive integrals can be computed with facility. This expression, however, is not so well adapted as that for a spiral, to the process of *summary* computation which becomes desirable when b is near r. The $(n+2x)^{th}$ term

$$Q_{n+2x} = Q_n \times \frac{4br^r}{t^4} \times \frac{n-1 \, . \, n+1 \, . \, . \, . \, n+2x-3}{n+2 \, . \, n+4 \, . \, . \, . \, n+2x};$$

whence, putting as before $\rho = \frac{4b^2r^2}{t^4}$,

$$\log Q_{n+2x} = Q_n + \log \rho \times x + \log\left(1 - \frac{3}{n+2}\right) + \log\left(1 - \frac{3}{n+4}\right) \ldots + \log\left(1 - \frac{3}{n+2x}\right).$$

Developing the logarithms, and stopping at $\frac{1}{n^3}$, this becomes

$$\log. Q_{n+2x} = \log. (Q_n \times \rho^x) - \text{modulus} \left\{ \frac{3x}{n} - \frac{3x}{2n^2}(2x-1) + \frac{x}{n^3}(4x^2 - 3x + 2) \right\},$$

which for $x = 10$ or 20 is sufficiently rapid.

The intermediate terms in this instance are more easily obtained by the method of quadratures, their sum being $\int_n^{n+2x} Q_n dx$. This process gives

$$S\{Q_{n+2} + Q_{n+4} \ldots + Q_{n+2x}\} = Q_n \left\{ A' - \frac{B'}{n} + \frac{C'}{n^2} \right\}, \tag{9}$$

in which

$$A' = x \pm \frac{x^2 . \log \rho}{2} + \frac{x^3 \log^2 \rho}{6} \pm \&c.$$

$$B' = \frac{x^2}{2} \{\pm 3 + 2x \log \rho\}$$

$$C' = \frac{x^2}{4} \{\mp 3 - 2x \log \rho + 10x\},$$

sufficiently exact, and easier for computation than their true values, $\frac{\rho^x - 1}{\log \rho}$, &c. In practice I found it best to take $x = -4$ and $+5$.

Having obtained the sum of any number of the terms Q, the sums of the preceding terms are successively obtained by the factors already given, and the multiplication must be continued till the products are certainly of an order that may be neglected.

If the sum of all these integrals $= S'$,

$$M = \frac{\mu F A}{\tan e} \times S' . z = \mu F A \times b\theta S',$$

and as $\theta = 2\pi \times$ number of spires in helix $(= s)$,

$$M = \mu F A \times 2\pi b S'. \tag{10}$$

The computation of S' is much facilitated by the terms Q containing only the inverse first power of u as a factor, so that when their sum is once got for any values of z, it is known for any other with a given b. The terms derived from Q are similarly computed in sum.

I have tabulated a few values of it, which will suffice to make an approximate comparison of this theory with observation.

TABLE II.

b.	$z = 1.$	$z = 2.$	$z = 3.$	$z = 4.$	$z = 5.$	$z = 6.$	$z = 7.$	$z = 8.$	$z = 9.$	$z = 10.$
1·130	2·1902	1·3456	0·9580	0·7367	0·5976	0·5017	0·4320	0·3791	0·3377	0·3044
1·564	1·9663	1·3382	0·9798	0·7670	0·6280	0·5294	0·4574	0·4023	0·3589	0·3242
1·998	1·4987	1·1642	0·8800	0·7052	0·5845	0·4973	0·4318	0·3811	0·3409	0·3081
2·432	1·3014	1·0418	0·8142	0·6516	0·5598	0·4817	0·4207	0·3727	0·3343	0·3042
2·866	1·1040	0·9206	0·7676	0·6443	0·5485	0·4749	0·4172	0·3712	0·3339	0·3031

For any point within or without the helix,

$$S = \frac{S'z' \pm S''z''}{z' + z''}.$$

It is useless to pursue the analytic part of the inquiry further at present, because the distribution of the magnetism excited by the spires in a closed circuit (which is quite a different problem from that of a magnetic bar) depends

on the law of induction from molecule to molecule, which is altogether unknown. Sir W. S. HARRIS has inferred, in the case of a permanent magnet inducting at a distance, that this law is the inverse of the distance. Without, however, inquiring how far his observations justify this conclusion, it manifestly cannot apply in the present instance, as the facts rather indicate an exponential function. If the coercive force of the metal did not interfere, the negative logarithm of the induction through iron should be proportional to the distance ; but the law of this force also is unknown.*

In examining the action of spires, and, still more, facts of induction, it is necessary to have a magnet of variable length, as no satisfactory conclusion can be drawn if several be employed, owing to the various qualities of iron. It is almost impossible to get two pieces of equal power, since the slightest difference in forging, turning, or planing, influences this property. That which was used in these experiments is of the same dimensions as the one described in my former paper, differing in being solid, and having its base of brass. The cylinders are connected by a slide, composed of two pieces of the same iron, each one and a quarter by two inches in section, in which semicircles are cut out to receive them ; and by steady-pins and screws it can be firmly attached at any height. From the excellence of the fitting the contact is very close ; and experience shows that it makes no interruption of the magnetic circuit. Setting it to leave four inches of each cylinder, I found that with 0·85 current force, and helices (F) containing 641 spires, the lift of the magnet is 817 lbs., when the screws are tight, and 800 when they are slackened, and the contact maintained by the attraction alone. If we allow for the decrease of magnetism mentioned in my first paper, these two may be considered identical. The sufficiency of the contact may also be inferred, from the parts of the cylinders below the slide showing no free magnetism. I found this to be the case even when this magnet was excited to the highest power which I have yet obtained with it.† I may add, that no part of the lifting power is due to the action of the spires on the keeper or slide : when the helices, even excited to the great power mentioned

* Bringing into contact with my magnet's N pole an iron tube, three-quarters of an inch in diameter, and nine feet long, seven feet of it are N, and the remaining two S.

† The cylinders were 2·1 long, the helices those already mentioned, and $\psi = 3005{\cdot}04$, $T = 81^\circ{\cdot}8$, L' was 1374·17.

in the note, were laid on the keeper in the same position as they had on the magnet, its extremities would neither attract a small key nor hold a horse-shoe keeper.

I have already pointed out the gradual decrease of the magnet's power during a series of experiments, and the fact that this decrease is prevented by continually reversing the current. On this plan most of the experiments whose results follow were conducted. It is made more effectual by exciting the magnet, and, without disturbing the keeper, suddenly reversing. It seems that the abrupt change of magnetic tension keeps the molecules of the iron in a state of neutrality, which prevents them from assuming permanent polarity.* To perform this easily, a commutator is attached to the magnet, and each set consists of six, half direct and half reverse. By this method the results have become far more consistent. The rest of the apparatus is unchanged except the rheometer, which is now on the construction discovered by M. GAUGAIN, and demonstrated by M. BRAVAIS.† Six rings, the largest 19^{i} in diameter, are placed parallel on a frustum of a cone, whose base is four times its axis: the centre of the needle is in the vertex of the cone, the needle is three inches long. This arrangement has the advantage of giving the proportionality of the force to the tangent of deflection—a far wider range than in the ordinary construction, and enabling to measure much higher currents. For such the largest ring alone is used; for small currents the whole six. A set of fifteen observations with the voltameter gives for its constants in the first case,

$$F = \tan\phi \times \log^{-1}(0{\cdot}58298)\,\{1 + \log^{-1}(6{\cdot}7607) \times (\sin^2\phi - \tfrac{3}{2}\sin^4\phi)\},$$

which will serve for any instrument of the same dimension.

In comparing the efficiency of different arrangements of spires or magnets, the most obvious method is to excite them till the lifts of the magnet are the same, when the *mean* efficiency of each spire, $= \mu$, must be inversely as ψ, the product of the force and number of spires. It would, however, involve an immense waste of time to ascertain this equality, and therefore it is better to refer them to a common standard. That which I have chosen is the action of

* This has been so effectual that the residual magnetism is insensible, though the number of times that it has been excited exceeds 1200.

† Comptes rendus, Jan., 1853.

a pair of spirals (A), possessing a definite character, and acting on the magnet under the most favourable circumstances, namely, when its cylinders are reduced so as merely to lodge the wire, in which case the action with a given ψ is the greatest possible. With these I obtained a series of values of L and the corresponding ψ', from which can be found, by interpolation, the ψ' corresponding to any L. If that L is obtained by any other spirals, helices, or altered length of the magnet, we have, assuming the mean efficiency of $(A) = 1$,

$$\mu = \frac{\psi'}{\psi}.$$

As, however, in using different currents, the magnet is unequally heated, it is necessary, in the first instance, to determine

THE TEMPERATURE CORRECTION.

In my former paper I investigated the correction by heating the magnets to about 70° and 170°; and, assuming that the decrease was uniform throughout this interval, I deduced the coefficient of decrease = 0·00033. The experiments of Dr. LLOYD on the temperature change of the magnetism induced by the earth on soft iron have led me to doubt the correctness of this assumption, and institute further experiments, which show that the law is much more complicated in the case of the electro-magnet, and that the coefficients which express it vary with the nature of the magnet. The magnet used in the first instance is one belonging to Mr. BERGIN'S collection (to whom I am indebted not only for the use of much valuable apparatus, but for still more valuable assistance in these researches), extremely convenient for the work. The cylinders are five and a half inches diameter, their centres six asunder; they are hollow, their thickness being half an inch; they are eight inches long, and the base and keeper have a section equal to theirs. The helices are those designated (I). The balance used with it is well worth notice. It has only one lever (whose ratio is 11·625), the longer arm of which carries a platform, on which weights, multiples of 30 lbs., can be placed: below the platform is suspended, by a spring balance, a tin vessel, into which shot is poured, whose weight is seen by the index up to 30 lbs., the limit of the spring. If this be not sufficient, the shot is permitted to escape by a valve at the bottom of the vessel, another weight is set on the platform, and the process is repeated. The

manipulation is easier than in mine, and the accuracy superior, but the concussion is greater. The hollow cylinders receive copper vessels, which may be filled with hot water or ice, and their magnitude is sufficient to preserve a nearly uniform temperature for some time. The experiments were, unfortunately, much disturbed by the perpetual passage of carriages through the street, which caused the loss of many results ; indeed we could only work during the night, and even then had much disturbance. The temperature was measured in the middle of the keeper, the middle of the base, and the top of the cylinders, and the mean taken. F was kept at 0·4734, giving $\psi = 143{\cdot}91$; T is the mean.

TABLE III.

	T.	L.	$O-C$.	No Obs.
1	41°·98	871°·83	− 22·68	6
2	53 ·56	876 ·48	+ 19·49	8
3	58 ·07	859 ·25	+ 12·85	6
4	64 ·78	842 ·23	+ 7·67	5
5	73 ·41	812 ·48	− 13·11	8
6	76 ·83	822 ·36	+ 1·42	9
7	81 ·85	823 ·61	+ 0 78	8
8	87 ·62	818 ·05	− 4·73	8
9	92 ·38	822 ·74	− 3·44	9
10	96 ·56	826 ·72	− 4·20	6
11	102 ·68	831 ·99	− 3·23	6
12	107 ·40	855 ·17	+ 13·98	6
13	111 ·89	854 ·83	+ 7·48	9
14	116 ·14	845 ·53	− 8·13	5
15	121 ·85	856 ·82	− 5·91	5
16	127 ·44	876 ·68	+ 4·74	8

The third column shows that the value of L diminishes as the temperature increases ; becomes a minimum at about 75° ; it then increases to the highest temperature in the Table ; after that it would certainly again decrease ; but we could not easily, with this arrangement, pass 127°. If there be no subsequent maxima and minima, these facts imply a formula such as

$$L = L' \{1 - at + bt^2 - ct^3\},$$

which, in fact, is the form given when the coefficients are determined by CAUCHY's method of interpretation.

By this, reckoning t from 60°, is derived

$$L = 842.54\{1 - t \times \log^{-1}(7\cdot35439) + t^2 \times \log^{-1}(5\cdot78313) - t^3 \times \log^{-1}(3\cdot46064)\}. \quad (11)$$

The fourth column contains the difference between the observed L and that calculated by this formula; the discordance is considerable, especially in the three first, but not greater than might be expected under the circumstances, and the errors being often effected with contrary signs shows that they are casual. I may also remark, that the current was not reversed in these observations. The difference between the correction given by this formula, and the change of L which I had obtained with the hollow two-inch magnet, showed the necessity of instituting similar experiments for that which I was using in the present series; and I found that by surrounding it and its slide with a covering of thick cloth, and the keeper with a similar one, I could raise the temperature to 220°, the limit of the thermometer which I used, by placing a gas flame on the base. The slide is the hottest; but its temperature, that of the cylinders at their top, and that of the keeper, were taken, and the mean deduced by giving each weight as the length of the piece.

The first was with the cylinders $= 0^{i}\cdot15$, and the spirals (A) with $\psi = 170\cdot79$. I obtained

TABLE IV.

No.	*T.*	*L.*	*O − C.*	No. Obs.
17	69°·6	615·90	+ 2·89	18
18	81 ·0	603·44	− 3·84	6
19	100 ·5	601·40	+ 0·94	12
20	130 ·0	598·89	+ 3·14	12
21	161 ·7	591·53	− 4·08	12
22	207 ·4	598·90	− 1·95	12

whence I similarly deduce

$$L = 618\cdot96\{1 - t.\log^{-1}(7\cdot03890) + t^2.\log^{-1}(4\cdot99503) - t^3.\log^{-1}(2\cdot43656)\}. \quad (12)$$

Secondly. With the cylinders $= 2^{i}\cdot1$, the helices (B) and $\psi = 553\cdot75$.

TABLE V.

No.	T.	L.	O – C.	No. Obs.
23	69°·1	902·36	0·00	16
24	96 ·1	888·98	+ 5·54	12
25	132 ·4	867·98	– 5·57	12
26	173 ·2	871·28	– 0·01	12
27	220 ·3	860·95	– 0·02	12

giving

$$L = 911{\cdot}85\ \{1 - t\,.\log^{-1}(7{\cdot}09736) + t^2.\log^{-1}(5{\cdot}08868) - t^3.\log^{-1}(2{\cdot}61666)\}. \qquad (13)$$

Thirdly. With the cylinders $= 10{\cdot}1$, with the helices (B), (J), (K), (L) and (M), containing 1002 spires, and with $\psi = 553{\cdot}75$, I obtained,*

TABLE VI.

No.	T.	L.	O – C.	No. Obs.
28	64°·4	729·45	– 2·25	12
29	97 ·5	707·34	+ 4·91	12
30	125 ·5	692·12	– 5·39	12
31	164 ·6	701·31	+ 0·50	12
32	208 ·6	690·43	– 0·01	12

giving

$$L = 735{\cdot}69\ \{1 - t\,.\log^{-1}(7{\cdot}30919) + t^2\,.\log^{-1}(5{\cdot}40470) - t^3\,.\log^{-1}(2{\cdot}98826)\}. \qquad (14)$$

It must be remembered, that these equations are mere formulæ of interpolation, and not the actual functions expressing the change due to temperature; yet there is an evident correspondence between them and the forces which are

* As the slide was at the bottom of the magnet, the plan of heating it, used for the others, was not available; but by placing a brass curved funnel over a double argand gas-burner, a stream of heated air was thrown within the covering of the magnet; and by placing its orifice so that part impinged on the slide, while the rest circulated within the confined space, I insured the same temperature in the slide and cylinders.

concerned. They all comprise three terms affected with t, and, of course, show a decrease at first to a minimum, then an increase to a maximum, and a subsequent decrease. Now, as I formerly noticed, the L represents the polarity at the contact surfaces of the magnet and keeper: this depends on, first, the polarization of those molecules which are excited by the helices; secondly, on that of the remainder of the magnetic circuit, and therefore its amount depends on the intensity of these polarizations, and on the facilities with which their influence is transmitted by induction. The intensity, again, depends on the susceptibility of the molecules directly, and inversely as the coercing force. Now each of these may be expected to change with the temperature. The correlation of heat with other molecular forces is such that, *à priori*, we would anticipate its lessening such forces as we are considering; and we find that at a red heat even iron is scarcely attracted by the most powerful magnets, which must depend on the molecules ceasing to be excitable. A diminution of this power, and of that which transmits the magnetism from one particle to another, must lessen L, while a contrary effect will arise from the diminution of the coercing force. All these influences will be functions of the lengths of the magnetic current, and of the excited cylinders; and accordingly we find that the coefficients in (12), (13), and (14), increase with the latter. Calling them α, β, and γ, and the length of the cylinder z, the three values of γ are exactly represented by the formula $a + bz$, and those of α nearly by $a' + b'^{z}$; from which one might infer that γ corresponds to that part of the change which belongs to the excitabiles, and a to the conduction. I have not found any simple expression for β, but since it gives the intermediate increase of magnetic attraction, which (as I hope to show in a future communication) depends on the coercive force, we may refer it to that.

I did not think it necessary to investigate these corrections for the other lengths of cylinder, as these three give sufficient data for interpolation, and from them,*

* Although it is not safe to interpolate beyond the limits of the observations, yet, computing from these the constants for twelve-inch cylinders, and reducing by them the temperature experiments with my first magnet, I find the higher gives in each pair an L' less than that of the lower by 7·56, 6·92, 4·44, 6·24, or in the mean 6·29 for a difference of temperature $= 101°{\cdot}6$. This, therefore, shows both that the same law holds in this magnet, though hollow, and that these constants will serve to reduce the observations made with it.

$$\alpha = 0{\cdot}001094 + \frac{z}{2} \times \log^{-1}(6{\cdot}17319) + \frac{z^2}{4} \times \log^{-1}(4{\cdot}90309).$$

$$\beta = 0{\cdot}00000989 + \frac{z}{2} \times \log^{-1}(4{\cdot}34242) + \frac{z^2}{4} \times \log^{-1}(3{\cdot}25527).$$

$$\gamma = 0{\cdot}00000002733 + \frac{z}{2} \times \log^{-1}(2{\cdot}14768).$$

The case is, however, very different if those parts of the circuït, which are not directly excited, be lengthened. With the cylinders = 10·1, and the helices (B), two inches high, placed in contact with the keeper, I got with $\psi = 544{\cdot}83$:

TABLE VII.

No.	$T.$	$L.$	$O-C.$	No. Obs.
33	64°·4	806·70	− 0·71	12
34	119 ·2	772·97	− 0·15	12
35	161 ·1	752·33	+ 0·25	12
36	197 ·9	732·81	− 0·08	12

These give

$$L = 810{\cdot}84\,\{1 - t\,.\log^{-1}(6{\cdot}99120) + t^2.\log^{-1}(4{\cdot}62797) - t^3.\log^{-1}(2{\cdot}21015)\}, \qquad (15)$$

when the coefficients are less than even in (12), but the law is the same as in the rest. The places of the minimum and maximum are much higher, and the change of L is greater than in any of the others.

The terms within the brackets in these expressions are, I think, independent of ψ; for in my former paper it is shown that, with the magnet there used, the decrease of L for a given difference of t is the same, with very considerable variations of the current.

ACTION OF SPIRALS.

I. In the first instance, I give the Table already referred to of the spirals (A), which I assume as my standard. Their constants are

$$b = 1{\cdot}14\ ;\ b' = 2{\cdot}80\ ;\ s = 40.$$

Their external diameter was intended to be as large as the distance of the cylinders would admit, and the cylinders themselves are shortened to 0·15.

giving the greatest number possible of spires, and the shortest magnet. The observed values of L are reduced to 60° by the coefficients of (12).

TABLE VIII.

No.	T.	Spiral.	L'.	$\Delta L'$	ψ'.	$\frac{\Delta\psi}{\Delta L'}$.	No. Obs.
37	68°·0	A.	696·96		201·49		12
				43.36		0·34503	
38	68 ·0	,,	653·60		186·55		12
				23·41		0·67322	
39	72 ·6	,,	630·19		170·79		12
				54·10		0·42218	
40	72 ·3	,,	576·09		147·95		12
				76·40		0·30301	
41	65 ·6	,,	499·69		124·80		18
				67·18		0·38538	
42	65 ·3	,,	432·51		98·97		12
				77·84		0·23523	
43	67 ·0	,,	354·67		80·66		12
				86·15		0·19900	
44	63 ·9	,,	272·51		64·31		12
				89·71		0·16252	
45	64 ·3	,,	182·80		49·73		12

As higher values could not easily be obtained without some risk of destroying the spirals by the evolved heat, I use, when necessary, the numbers of Table XII. obtained with the helices B, whose ratio to A is known.

II. We can now compare spirals of different diameters, and thus ascertain how far the preceding theory agrees with experiments. The cylinders were set to $0^{i}\cdot 6$, which permits the use of spirals $7^{i}\cdot 5$ diameter by overlapping them. The spirals are made of flatted copper wire 0·2 by 0·05, to enable the employment of powerful currents with such batteries as I possess;* but experience makes me regret this arrangement, for it requires a greater length of cylinder, and the action of the current is probably not quite uniform

* The current employed with (C) would evolve in a voltameter 18 cubic inches of mixed gases per minute.

through the section of the wire. The error, however, must be trifling. Their constants are

(C)	$b = 1{\cdot}135.$	$b' = 1{\cdot}905.$	$s = 18.$
(D)	„ $1{\cdot}130.$	„ $2{\cdot}770.$	„ $40.$
(E)	„ $1{\cdot}150.$	„ $3{\cdot}873.$	„ $60.$

I obtained with them

TABLE IX.

No.	Spiral.	T.	L'.	ψ.	ψ'.	μ.	Theoretic μ.	No. Obs.
46	(C)	69°·6	641·86	187·32	178·63	0·9684	1·3772	12
47	(D)	66 ·0	598·85	188·02	156·12	0·8303	1·0000	12
48	(E)	66 ·2	606·07	193·78	158·27	0·8167	0·8229	12

L is reduced by (12), as from the shortness of the cylinders it must nearly be exact for them also. These results are in a ratio so different from what I anticipated, that I suspected some error was produced by the overlapping of the spirals E. I therefore repeated the experiments with one of Mr. Bergin's magnets, in which the distance of the axes is 7·5, and the length of the cylinder $0^{i}{\cdot}5$; but it gave similarly

TABLE X.

No.	Spiral.	T.	L.	ψ.	ψ'.	μ.	Theoretic μ.	No. Obs.
49	(C)	63°·6	502·32	193·38	125·69	0·6497	1·3772	12
50	(D)	64 ·3	457·56	189·14	107·53	0·5685	1·0000	12
51	(E)	63 ·2	458·38	191·74	107·84	0·5624	0·8229	12

Both sets agree in showing that the power of a spire does not decrease nearly so fast by an increase of diameter as the equation (7) assigns. In both the μ is diminished relatively to that of A by the greater length of the cylinder, and (in the second) of the entire magnet and its keeper, as will be more evidently shown hereafter. As, however, (D) is almost identical in its diameter with A, by taking it as unit, we get a more distinct comparison of the relative values of μ.

TABLE XI.

	Tab. IX.	Tab. X.	Theoretical.
C	1·1663	1·1428	1·3772
D	1·0000	1·0000	1·0000
E	0·9836	0·9893	0·8229

The formula gives more power to the spires of C, less to those of the zones $D-C$ and $E-D$ than is observed: if, in fact, we determine what parts of the effect of E belong to them, we find for

$$C \text{ effect} = 10{\cdot}4967 = 9 \times 1{\cdot}1663.$$
$$D-C \quad \text{,,} \quad 9{\cdot}5033 = 10 \times 0{\cdot}9503.$$
$$E-D \quad \text{,,} \quad 9{\cdot}5080 = 10 \times 0{\cdot}9508.$$

It appears *probable*, from Table I., that the two or three innermost spires act much more powerfully than the rest; and therefore it seems that, from them to a considerable distance, *the exciting force of the others is constant.* The discrepancy between the computed and observed μ is far too great to be caused by error of observation; for again taking D as the standard, the theoretic μ will give L' 48 greater for C, and 61 less for E, than the true values, while the probable errors of the latter are only 2·04 and 1·99.

ACTION OF HELICES.

Here also the discrepancy between theory and observation is considerable, independent of the interference of induction. This is shown, even without measures, by some striking facts; for instance, the helices (F) being placed above the polar surfaces of the cylinders set to 4·1, but separated by plates of zinc $\frac{1}{32}$ thick, and excited to have $\psi = 552$, the magnetism produced was scarcely sensible when a keeper was applied across the cylinder, immediately below them; I had no means of measuring it, but its attraction was not more than a pound or two. Had the helices been on the cylinders, L would be 850. If one of these helices be placed in the same way above one polar surface, there is scarcely any attractive power developed at the other: in this case, however, as the magnetic circuit is incomplete, the force is much less; but I expected to find 30 or 40 pounds at least. Similar results were obtained with a magnet, the upper two

inches of whose cylinders are iron, the rest brass. When the helices were on the brass, just below the iron, the lift was but 0·18 of its amount when they were on the iron. At the extremity of the helix, the exciting force might be supposed to be little less than in its interior. The constants of the helices which I used are

(B)	$b = 1·130$.	$b' = 1·965$.	$z = 1·8$.	No. layers = 8.	$s = 214$.
(F)	„ 1·130.	„ 2·866.	„ 1·8.	„ 20.	„ 641.
(G)	„ 1·110.	„ 1·210.	„ 7·6.	„ 2.	„ 324.
(H)	„ 1·650.	„ 1·750.	„ 7·6.	„ 2.	„ 320.
(I)	„ 2·890.	„ 2·990.	„ 7·6.	„ 2.	„ 304.
(J)	The same dimensions as (B).			„ 8.	„ 192.
(K)				„ 8.	„ 188.
(L)				„ 8.	„ 183.
(M)				„ 8.	„ 225.

(G), (H), and (I) are from Mr. Bergin's collection; the two last were kept concentric with the magnet by wooden framing, though a considerable error in this respect seems to have little effect.

The results obtained with (B) are given in a separate Table, as they were intended to be used for interpolation beyond the range of Table VIII., and, therefore, the first differences are included in it. L' is reduced by (13), the cylinders being 2·1.

TABLE XII.

No.	T.	L'.	$\Delta L'$.	ψ.	$\frac{\Delta\psi}{\Delta L'}$.	No. Obs.
52	68°·1	1098·90		1059·35		12
			63·47		4·02143	
53	69 ·1	1035·43		804·11		12
			124·55		2·01991	
23	69 ·2	910·88		552·53		16
			96·25		1·36820	
54	64 ·4	814·63		420·84		12
			111·22		1·25265	
55	64 ·2	703·41		281·52		12
			114·74		0·70865	
56	65 ·4	588·67		200·21		12
			110·80		0·44052	
57	62 ·6	477·87		151·40		12
			145·03		0·34793	
58	63 ·8	332·84		100·94		12

The four last are comparable to Table VIII., and give for μ,

$$\begin{array}{r} 0·7571 \\ 0·7606 \\ 0·7577 \\ 0·7498 \\ \hline \text{mean} = \log^{-1}(9·87912) = 0·7570 \end{array}$$

The others are given in

TABLE XIII.

No.	Helices.	T.	L'.	ψ.	ψ'.	μ.	Calc. μ.	Cylinder.	No. Obs.
55, 58	} *B*)	° . . .	. . . .	. . . .	. . . .	0·7570	0·8288	2·1	48
59	(*F*)	71·4	874·96	550·72	380·42	0·6927	0·7038	2·1	10
60	(*B* + *J*)	67·0	848·91	544·20	353·21	0·6490	0·5452	4·1	11
61	60 + (*K*)	66·9	810·70	547·78	318·61	0·5816	0·4073	6·1	17
62	(*G*)	63·5	767·41	547·19	272·53	0·4981	0·3280	8·1	11
63	(*H*)	63·2	747·34	541·89	249·41	0·4603	0·3211	8·1	11
64	(*I*)	61·6	737·58	543·49	240·74	0·4430	0·2695	8·1	11
65	61 + (*L*)	67·4	765·12	547·65	270·35	0·4937	0·3228	8·1	11
28	65 + (*M*)	64·4	733·44	553·75	237·48	0·4289	0·2552	10·1	12
33	*B*	64·4	810·13	544·83	314·12	0·5765	. . . .	10·1	12
66	*F*	71·7	797·39	548·04	301·56	0·5502	. . . .	10·1	10

Here also the difference between the theory and observation is considerable; more so than appears at first sight. The computed values of μ given in the eighth column assume that the intensity in every part of the magnetic circle is equal; or, in fact, that magnetism is transmitted by induction without any diminution. That this is not the case is evident from comparing Nos. 55, 58, and 33, in which the difference of sixteen inches in the two cylinders of the magnet reduces the actual efficiency of the spires of B from 0·757 to 0·5765. Therefore, all these calculated values are much too great, and yet all of them, except the first, are less than those given by observation, while in the spirals they are greater.

The decrease of efficiency depending on an increase of the spire's diameter is less than that assigned by theory, still more than in the spiral. In (F) and (B), which are nearly of the same radii as (D) and (C), the ratio of μ is 0·915

with the cylinders 2·1, and 0·954 with 10·1 (which case does not properly belong to this part of my subject, but is given for the comparison), instead of 0·849.

This discrepancy is still more evident with the cylinder 8·1, where (G), (H), and (I) are mere cylindric annuli, but where the ratio of (I) to (G) is found to be 0·8894 instead of 0·8217.

The same is the case with respect to the influence of the cylinder's length; as is shown in the following Table of the ratio of the observed to the theoretic μ, for helices equiradial to (B); in which I have also given it for the spirals and three helices.

Table XIV.

Cylinder.	$\frac{\text{Obs. }\mu}{\text{Calc. }\mu}$ Helices.	Mean Diameter of Spiral.	$\frac{\text{Obs. }\mu}{\text{Calc. }\mu}$ Spiral.
0·6	0·7031	C 1·520	0·7031
2·1	0·9123	D 1·700	0·8303
4·1	1·1904	E 2·511	0·9925
6·1	1·4279	G 1·16	1·5221
8·1	1·5294	H 1·70	1·4335
10·1	1·6807	I 2·94	1·6438

This divergence was not expected by me; for the principles on which the equation (1) is founded have been found to give correctly the action of helical currents on each other, and their deflection of a magnetic needle. There is, however, one marked difference between these and the case of the electro-magnet. The polarities of two currents cannot be in any way altered by their mutual action; those of the molecules of the needle are kept nearly permanent, both in intensity and direction, by the coercive force of the hard steel; so that the ordinary methods of statics apply with certainty to them. In the case of soft iron, both these vary with the condition of the current, and according to laws which I do not think are fully known. From the excess of activity of the outer and lower spires, I am inclined to suspect that the resolution of the exciting power in the direction of x and z is the main cause of error, though some of it, as I have already indicated, must also belong to the mutual induction of the molecules.

Some practical inferences may be drawn from these experiments, for the construction of electro-magnets intended to act with a closed magnetic circuit.

1. The nearer the spires can be kept to the polar surfaces, the better, for their activity is much diminished as they recede from it: the great superiority of spirals over helices shows this. Thus, the efficiency of (A) is 1·4436 times that of the equiradial (F); their mean distances from the poles being 0·07 and 1·07.

2. The very small decrease caused by increasing the diameter (at least, as far as 7·5, and probably beyond it to an extent not likely to occur in practice) leads to the conclusion that the helices should be as wide as the distance of the cylinders permits, or that b' *shall be half that distance*; b will, of course, be as nearly as possible the radius of the cylinders.

3. The height of the helices and cylinders should be as little as is consistent with lodging a sufficiency of wire to employ to the best advantage the power of the battery which is used.

4. This height $= z$ may be determined thus :—

Let E and R be the constants of the batteries; $d =$ the diameter of the wire; $d + c$ that of it when covered with thread (c being in my wire $= 0·03$); $2s =$ the number of spires in the helices; and ρ' their resistance. We have

$$2s = \frac{2z(b'-b)}{(d+c)^2};\ \rho' = \frac{8\rho\,.\,(b'+b)(b'-b)\times z}{d^2(d+c)^2},$$

ρ being a constant, such that the resistance of a unit of the wire $= \frac{4\rho}{\pi d^2}$: for copper I find $\rho = \log^{-1}(5·71018)$. Then we have for the exciting power with a unit current,

$$\mu\psi = \frac{2s\mu E}{R+\rho'} = \frac{2E(b'-b)}{R} \times \frac{\mu z}{(d+c)^2 + \frac{8\rho z(b'+b)(b'-b)}{Rd^2}};$$

or, putting $a = \frac{8\rho(b'+b)(b'-b)}{R}$,

$$\mu\psi = \frac{2E(b'-b)}{R} \times \frac{\mu z}{(d+c)^2 + \frac{az}{d^2}}.$$

If in this we consider d and z as variables, μ a function of z, and differentiate for the maximum, we obtain the equations,

$$o = d^3(d+c) - az,$$
$$o = \frac{zd\mu}{dz}\left\{1 + \frac{az}{d^2(d+c)^2}\right\} + \mu. \qquad (16)$$

Substituting, in the second, for az its value in (16),

$$o = \frac{zd\mu}{dz}\left\{1 + \frac{d}{d+c}\right\} + \mu. \qquad (17)$$

When z is determined by any particular condition, (16) gives the most advantageous diameter of wire, and *vice versâ.*

If it be not, and if the relation between it and μ be known, the two equations give the d and z for the absolute maximum. In the case of my magnet (when $b' = 3$, $b = 1{\cdot}13$) Nos. 55, 58, 60, 61, 65, and 28, give the means of expressing that relation by an interpolation formula, $A - Bz + Cz^2 - Dz^3$. Supposing the battery to consist of ten Groves' such as I use, $R = 47$, with these I obtain

$z = 8{\cdot}39$,
$d = 0{\cdot}14725$, or nearly No. 9 of the wire gauge,
$s = 999$,
$\mu\psi = 2780{\cdot}29$.

A much higher power, however, would be obtained if the ten cells were grouped as five double cells, and the helices made to suit this condition. In this case $R = 11{\cdot}75$; and we find for the best arrangement,

$z = 8{\cdot}33$,
$d = 0{\cdot}2106$, a little more than No. 6,
$s = 538{\cdot}18$,
$\mu\psi = 3549{\cdot}60$.

5. I suppose the current to traverse the helices consecutively; but they are frequently used collaterally with the notion of obtaining a more powerful current. This is not to be recommended in ordinary cases. If the numbers of the spires be s' and s'', and their resistances ρ' and ρ'',

$$\text{consecutive } \psi : \text{collateral } \psi_{,} :: s'+s'' : \frac{s'\rho''+s''\rho}{\rho'+\rho''} \times \frac{R+\rho'+\rho''}{R+\frac{\rho'\rho''}{\rho'+\rho''}}.$$

If, as is usual, $s'=s''$, $\rho'=\rho''$, the ratio becomes

$$\psi : \psi_{,} :: 2R+\rho' : R+2\rho'.$$

When ρ' is greater than R the collateral arrangement is best, but then it must be observed, the expenditure of zinc and acid is twice as great as in the ordinary arrangement.

In these experiments the cylinders were entirely covered with spires; when this is not the case, and particularly when the helices are at a distance from the polar surfaces, the action is diminished, because it is imperfectly transmitted by induction. Such observations seem calculated to advance our knowledge respecting that power, and I hope soon to submit to the Academy those which I have made with this view; and some respecting electro-magnets of hard steel and cast iron.

ERRATA.

Page 503, in (5), and the preceding equation, *for* + *read* ×.
" 504, Table I., line 2, *for* ·47328 *read* 1·47328.
" 507, in (10), *after* S' *read* s.
" — line 15, *for* it *read* $2\pi bS'$.
" 511, line 1 from bottom, *for* interpretation *read* interpolation.
" 514, line 23, *for* excitables *read* excitability.
" 515, line 10, *for* when *read* where.